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Visualizing Vastness

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This is the final essay in a six-part series.

In the funky, crunchy, slightly gritty college town where I live, we have a pedestrian mall called the Ithaca Commons. You can probably picture it: A gem store. A hemp shop. Lots of places to buy hand-made candles.

And a scale model of the solar system five billion times smaller than the real thing.

Built in honor of Carl Sagan, the Cornell astronomer, author and science communicator, the Sagan Planet Walk offers lessons that reach far beyond astronomy. It's a case study in visualizing vastness.

Admit it. You have no real feeling for the size of the solar system. That's O.K. Nobody else does either. Even knowing the numbers doesn't help much. If I tell you the Earth is about 8,000 miles in diameter and 93,000,000 miles from the Sun, does that give you any sense of the distances involved? No, because the numbers are too big. Things that are so far removed from our daily experience - like quarks, and dinosaurs, and Kim Kardashian - are inherently hard to understand.

The designers of the Sagan Walk made the solar system accessible by shrinking it to a human scale. Each planet is displayed in its own monolith.

As you stroll from one to another, you can't help noticing that the first four planets are really close together. It takes a few seconds, a few tens of steps, to walk from the Sun to Mercury and then on to Venus, Earth and Mars. By contrast, Jupiter is a full two-minute walk down the block, just past Moosewood Restaurant, waiting for someone to stop by and admire it. The remaining planets are even lonelier, each marooned in its own part of town. The whole walk, from the Sun to Pluto, is about three-quarters of a mile long and takes about 15 minutes.

The planets themselves are scaled down too, in exact proportion. The tiniest ones, Mercury and Pluto, look like little grains of couscous. The Earth resembles a pea. The largest ones, Jupiter and Saturn, are the size of donut holes. The Sun is about 10 times wider still, the diameter of a serving plate.

The Sagan Walk is the ultimate in egocentric fantasies: it centers the solar system on *you*. And I don't mean that figuratively; it's literally true, in a certain numerical sense.

To see what I mean, let's begin with the basics of handling large numbers. The key is to avoid them where possible; try to work with numbers close to 1 instead.

For example, you wouldn't state your height in billions of nanometers. It's not that it wouldn't be correct; it just seems silly and it would be hard to work with, because people aren't very good at comprehending numbers like a billion. The right scale for human height is meters, not nanometers.

Or think about currencies where even the most insignificant trinket costs millions of lira. It's annoying and confusing. That's because one lira has ceased to be the right scale for that currency. They should be using mega-lira, not lira (as Turkey did in 2005 with the introduction of the "New Turkish Lira," defined to equal 1 million old liras).

Another helpful tactic is to express numbers in "scientific notation." That way of writing them highlights their most essential features and shunts their minor details aside.

For example, take the number 1,234. Round it down to the nearest power of 10, which is 1,000. Then write 1,234 = 1.234 x 1,000. Count the zeros (in this case, 3); pop that number into an exponent to get the corresponding power of 10 (here, 10^3), and stick whatever is left over out in front (1.234). The result is 1,234 expressed in scientific notation: 1.234 x 10^3. Its "order of magnitude" is said to be 3, because that's how many powers of 10 are in it.

Why bother? Because scientific notation teases out what's important (the powers of 10) from what's less important (the leftovers like 1.234, which are always less than 10, by definition, and in that sense close to 1). When you're trying to make rough estimates, the powers of 10 are what matter. They give the lion's share of the answer.

Now let's apply these ideas to the solar system. What happens if we shrink all the pertinent distances and diameters by a factor of five billion, as the designers of the Sagan Walk did?

Consider the longest length scales, the distances from the Sun to the farthest planets. After the five-billion-fold reduction, those previously unfathomable distances shrink to being around a kilometer, which - in terms of all-important you - is roughly a thousand times longer than the one-meter scale of your body. That's 10 x 10 x 10 times longer than you. Three powers of 10.

Next, look what happens to the shortest length scales, the diameters of the tiniest planets like Mercury and Pluto. They become millimeter-size, or about a thousand times *shorter* than you. Another three powers of 10, but in the opposite direction.

Which puts you right in middle. You are now the 1, the measure of all things.

The Sagan Walk is a remarkable achievement in the visualization of vastness. It succeeds because its designers chose the right reduction factor *and* because the six orders of magnitude in the solar system happen to be within our perceptual power to grasp all at once.

But for many other problems that scientists study, no single reduction or enlargement will suffice. A good strategy in such "multiscale" cases is to use a series of reductions and enlargements that progress smoothly from one to the next.

This was the approach taken by Charles and Ray Eames, the husband-and-wife design team who gave us the iconic short film "Powers of Ten." They centered their view of the universe on a couple enjoying a lazy picnic in the park.

Instead of the single scale factor used in the Sagan Walk, "Powers of Ten" uses a continuous zoom - a visualization technique that seems commonplace today, but which blew the minds of its audience in 1968.

The movie zooms through 40 powers of 10, starting from the picnickers, then pans back to the largest scales of the known universe, and finally reverses direction in a dizzying descent down to the sub-nuclear scale of quarks. The genius of the movie is that as time ticks by *linearly*, like 1, 2, 3, 4, the field of view contracts or expands *exponentially*, changing tenfold at each step from 10 meters to 100 to 1,000 to 10,000. In effect, the counting takes place in the exponent - 10^1, 10^2, 10^3, 10^4 - and not in the number itself.

This style of thinking, this powers-of-10 mentality, is our best hope for making sense of the immensity of the natural world. What makes subjects like biology and climate science so hard is not just that they involve so many variables; it's that the crucial phenomena in them occur over such a wide range of scales. Biologists need to contend with everything from nano-size DNA molecules on up to cells, organs, organisms and ecosystems. For climate scientists the relevant scales go from the molecular (the photochemistry of ozone) to the global (the fluid mechanics of the jet stream). Many of the great scientific puzzles of our time have this multiscale character.

A contentious example, especially in this election season, is inequality. The distribution of wealth in the United States spans at least 10 powers of 10, ranging from people whose net worth is measured in tens of billions of dollars, to those with barely a dollar to their names. This disparity dwarfs even the six powers of 10 in the solar system. As such, the distribution is extremely difficult to depict on a single graph, at least on the standard kinds of plots with linear axes, which is why you never see it displayed on one page.

Depending on your politics, you may think that wealth inequality is a problem to be solved, or irrelevant, or an encouraging sign of a free society. But whether you believe we need more inequality or less, I think we can all agree that it would be helpful to understand the actual distribution. Unfortunately its multiscale character confounds us.

This is clear from the work of Michael I. Norton and Dan Ariely. In 2005 they surveyed a representative sample of more than 5,500 Americans - men and women, rich and poor, conservative and liberal, young and old - and asked them two questions: How much wealth inequality is there in America? And how much should there be, ideally?

Norton and Ariely found that people on both sides of the political spectrum grossly underestimated the extent of inequality. The typical respondent believed that the top 20 percent owned 59 percent of the nation's wealth, much less than the 84 percent the top quintile actually owned (at the time of the survey). Respondents also thought the two quintiles at the bottom - the poorest 40 percent - owned 10 percent of the nation's wealth, when the reality was that their two slices totaled 0.3 percent of the American pie, the two nearly invisible slivers in the chart.

Yet surprisingly, when asked to describe the ideal distribution they'd *like* to see, respondents of all ages, classes, genders and party affiliations agreed. They'd all prefer a distribution much less extreme than the status quo: the top quintile would hold about 32 percent of the wealth, while the poorest quintile would have over 10 percent.

It's nice we can all agree about something for once, even if it happens to be a more equal distribution of wealth than exists in any country on Earth and probably in our solar system.

NOTES

1. The Carl Sagan Planet Walk is truly awe-inspiring. Come visit us here in Ithaca, N.Y., and see for yourself. But if you can't make the trip, this video will give you an impression of what it's like. For more information, including pictures, maps and history, check out the Sagan Walk's official Web page and this blog post by J.W. Ocker.

A new station was added to the Sagan Walk on Sep. 28, 2012. Representing Alpha Centauri - the nearest star to the sun - it is located at the Imiloa Astronomy Center on the University of Hawaii's Hilo campus. The Sagan Walk now measures 5,000 miles from end to end, making it the world's largest exhibition.

For those unfamiliar with Carl Sagan, start by listening to his meditation on a photograph of Earth taken by the Voyager spacecraft as it looked back from Saturn, 4 billion miles away, in which our planet appears as a single pixel, a "pale blue dot." His words will move you.

2. The federal budget is another important topic that bewilders most of us because of the gigantic numbers involved. In an illuminating blog post, the mathematician Terry Tao brings those numbers down to size by converting them to their household equivalents, using a conversion factor of 100 million to 3. Thus, when you hear about the government collecting or spending $100 million, think of it as $3 in family terms. Tao credits the idea for this rescaling to an observation made by the economist Greg Mankiw. In 2009, when President Obama called for a $100 million cut by federal agencies as a sign of fiscal discipline, Mankiw noted that this would be like a family with an annual spending of $100,000 and a budget shortfall of $34,000 deciding to cut $3, "approximately the cost of one latte at Starbucks."

3. The "Powers of Ten" Web site contains more information about this classic film, as well as interactive tools that allow you to zoom through the scales of the universe at your own pace. Also, the superb PBS documentary "Charles and Ray Eames: The Architect and the Painter" offers a revealing portrait of the creative couple behind "Powers of Ten."

4. Parents and children will find much to explore at this Web site about the basics of scale models, ratios and scale factors.

5. For the survey of what Americans think about wealth inequality, see M. I. Norton and D. Ariely, "Building a better America - one wealth quintile at a time," Perspectives on Psychological Science, Vol. 6, No. 1 (2011), pp. 9-12, and Dan Ariely's recent piece in The Atlantic. The wealth inequality data they used are available in a working paper by Edward N. Wolff. Summary statistics for the wealth distributions in other countries are tabulated here.

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